

Failure Analysis of a Pole Gin

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Lugs on a cast aluminum/fiberglass pole gin failed while raising an electrical transformer on a power pole. This same system had just lowered a heavier transformer. The pole gin consisted of a cast aluminum base that was strapped to a utility pole by a nylon belt-and-ratchet mechanism. A fiberglass pole was mounted in the base, and a pulley was attached to the other end through another aluminum casting. Rigging for the lift was complex and required a physical simulation to estimate actual lug hole loads and to determine that overall loading was within the manufacturer's published limits. Possible abuse by hammer blows was evaluated by dynamic testing to measure force attenuation in the system. Results ruled out abuse as a factor. Literature revealed that the heat treatable Precedent 71A, or A771-T7 alloy, used for the base casting was very susceptible to stress-corrosion cracking (SCC). Evidence was observed for features indicative of creep-rupture damage on the fracture surface. Evaluation of all of the evidence led to the conclusion that time-dependent crack growth, most likely by both SCC and creep-rupture, plus the effect of bolt hole loading on crack growth could best explain the failure of the gin under a less severe condition than had just occurred earlier in the day.

Keywords: aluminum casting, dynamic mechanical analysis, failure analysis

Introduction

Routine maintenance for electrical utilities can involve changing out transformers on power poles. Line trucks equipped with hydraulic boom cranes are normally used for this operation. However, poor access in residential areas may render use of a line truck difficult or impossible. A variation of a gin-pole-type derrick attached to the pole itself is used in these circumstances. The pole gin used for these applications is a fixed boom with a ring for a pulley on one end and a means of attaching the pole gin to the pole on the other.

Pole gins have been in use for many years and have traditionally been made from steel (based on interviews with power companies). The gin in this accident was a new design combination of a cast aluminum base (771-T7) and end fitting (Almag 35) with a large fiberglass rod in between to form the gin pole. A nylon belt with a ratchet was attached to the base through a bolt-and-lug design. The gin had been in service for approximately two years when lugs holding the ratchet fractured while raising a transformer. The belt and ratchet flew back and wrapped around the metal cross brace for the wooden beam carrying secondary wires. These wires were

pulled down by the weight of the still-attached transformer and onto two linemen on the pole, electrocuting one and injuring the other.

Linemen had just successfully lowered a 295 kg (655 lb) transformer and were in the process of raising a 274 kg (604 lb) transformer when the

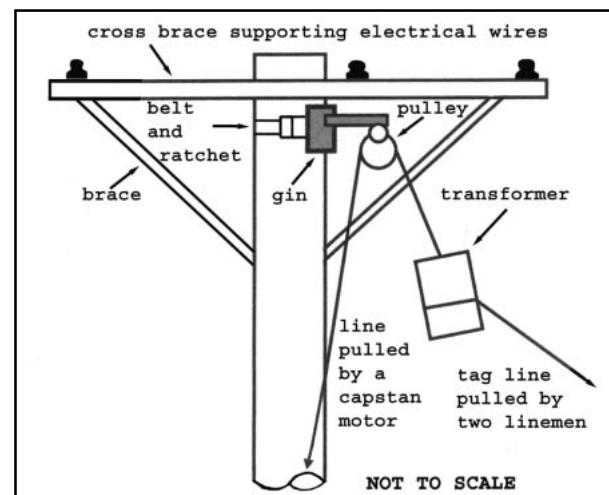


Fig. 1 Schematic arrangement of rigging for transformer replacement. The complexity of the rigging required experimental procedures to measure forces on the gin and lugs.



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accident occurred. Rigging consisted of a single pulley attached to the pole gin end fitting (Fig. 1). A slow-speed capstan at the base of the pole powered the lifting rope. Phone lines on the side of the pole and just below the transformer required using a tag line, with two men on the ground to move the transformers over the lines. The pole gin was rated for 8900 N (2000 lb), with no restriction on loading direction.

This paper provides the results of an extensive analysis of the pole gin failure. Lifting accidents always raise the issue of overloading. Full-scale tests were run to estimate actual loads on the lugs holding the ratchet to the base and to determine whether the overall loading was within the manufacturer's stated 8900 N (2000 lb) limit. These tests also confirmed that the transformer could be lifted as the linemen had testified. Hammer marks were observed on the end fitting for the pulley. Dynamic force measurements were made on an exemplar pole gin to determine whether hammer blows on the end casting (to adjust the gin position) could have damaged the lugs.

Fractographic Analysis

Figure 2 shows the overall gin, with two broken lugs where the 12.7 mm (0.500 in.) diameter bolt holding the ratchet pulled out. Figure 3 shows a scanning electron microscope (SEM) micrograph of the fracture surface at the hole wall in the upper lug. Intergranular fracture and microshrinkage can be seen up to the hole wall. No evidence of cyclic fatigue could be observed in any of the hole wall areas on either side of the hole wall in the upper lug.

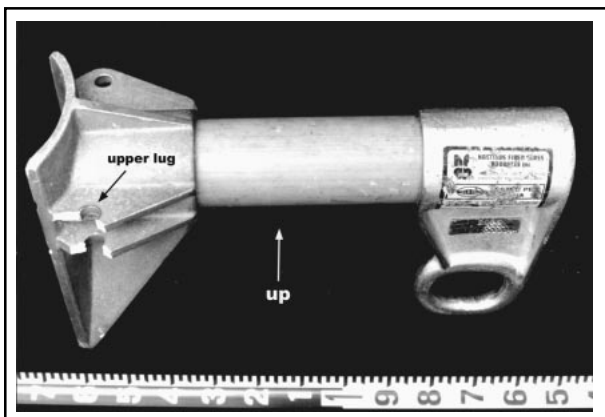


Fig. 2 Two broken lugs where ratchet attached to base of pole gin

Evidence of intergranular fracture, microshrinkage, and triple-point-type grain-boundary voids are shown in Fig. 3 and 4. These features were characteristics of the fracture surfaces in both lugs on the gin. Intergranular fracture is evidence for a weakness in the grain boundaries, usually resulting from segregation of elements to the boundary surfaces. A precipitate pattern indicative of grain-boundary precipitation can be seen on grain-boundary facet surfaces in Fig. 5. A triple-point void (void at the juncture of three or more grains) can also be seen in Fig. 5. This feature is usually associated with creep-rupture failure mechanisms. Interdendritic porosity or microshrinkage is very obvious in Fig. 3 and 4. The triple-point void in Fig. 5 is not characteristic of microshrinkage. Dated and out-of-print company

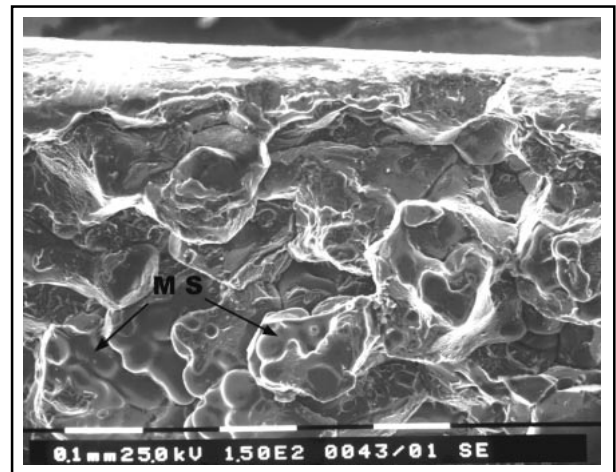


Fig. 3 Intergranular fracture and microshrinkage observed at hole wall area on upper lug fracture

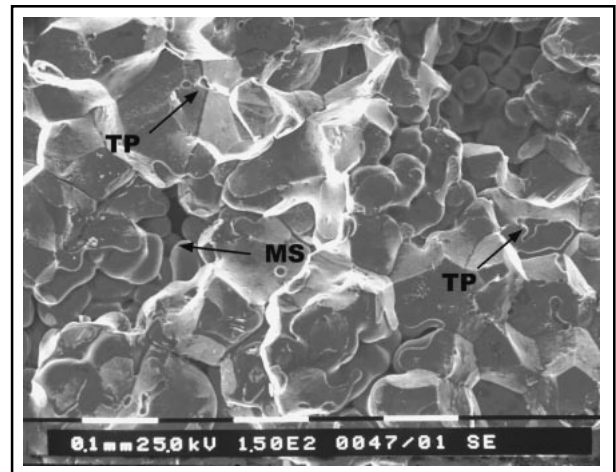


Fig. 4 Fracture area further in from origin area of Fig. 2 also shows intergranular fracture, microshrinkage (MS), and triple-point (TP) grain-boundary voids.

product literature warned of creep at room temperature at high stress levels with this alloy. Unpublished research by the author in the early 1970s on overload retardation in crack growth clearly revealed rapid room-temperature stress relaxation in 7075-T6 alloys. Therefore, it is quite possible that the triple-point void in Fig. 5 is actually a manifestation of a creep-rupture process in this alloy.

Careful study of the fracture surfaces revealed a large extent of voids. Point-counting areas that showed no evidence of a fracture mechanism on SEM micrographs yielded projected surface void area estimates of 26 to 47%. Void area percentages on polished specimens cut just below the fracture yielded approximately 6% void area. This high-percentage void area on the fracture surface can

provide a reason for locally high stresses leading to creep-rupture damage.

Metallographic Analysis

Specimens for metallography were cut just below and parallel to the fracture surfaces of the outboard lug cracks. Polishing was carried out with diamond abrasives on a napless cloth to reduce the risk of inclusion pullout. Figure 6 shows the two different types of grain-boundary precipitates that can be seen when using Keller's reagent. Sodium hydroxide etchant provided somewhat better resolution of the grain-boundary precipitation, as shown in Fig. 7. Nearly continuous grain-boundary voids are shown in Fig. 8. This is consistent with the observation of extensive void surface on the fracture.

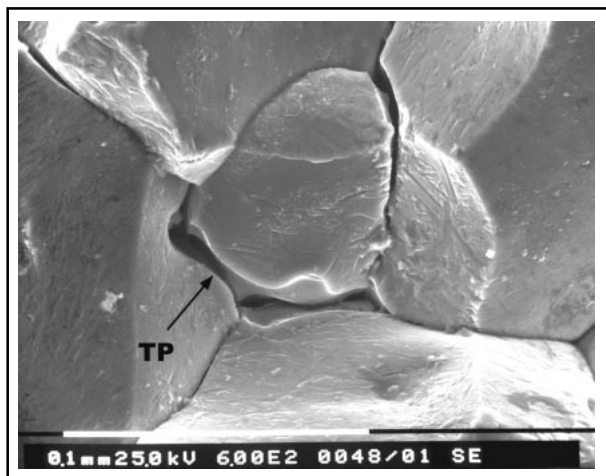


Fig. 5 Faint patterns on grain-boundary faces indicate grain-boundary precipitates.

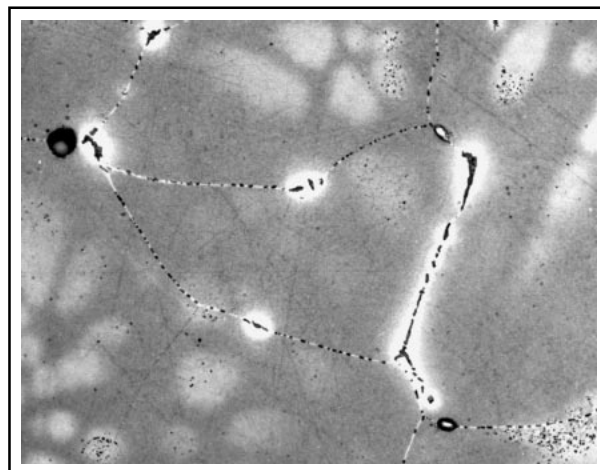


Fig. 7 A 2% sodium hydroxide etch revealed more detail of the grain-boundary precipitation.

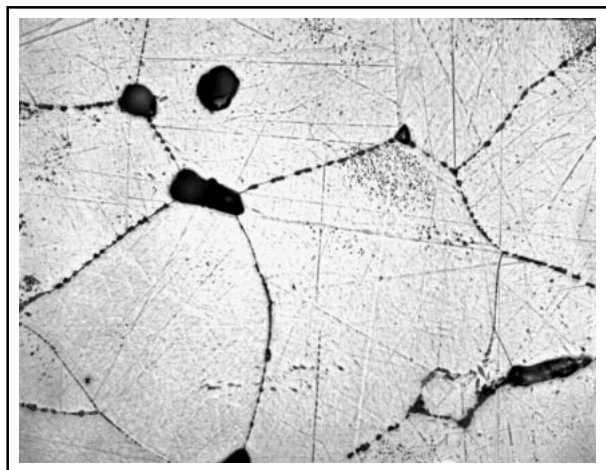


Fig. 6 Large grain-boundary precipitates in upper left corner. Fine grain-boundary precipitates are present in most of the boundaries. (Keller's etch)

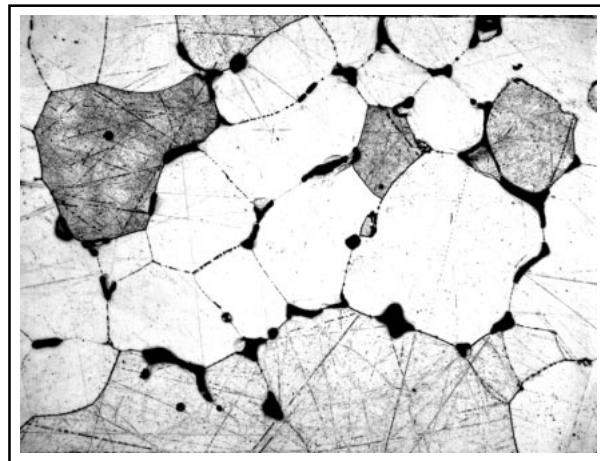


Fig. 8 Nearly continuous grain-boundary voids. This is consistent with the extensive grain-boundary voids observed on the fracture surfaces. (Keller's etch)



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Cracked Exemplar

A second cracked pole gin was discovered during inspection of a group of gins returned from service. A crack in the outboard side of the upper lug hole wall could be seen visually (Fig. 9). A second crack propagating toward the base was not apparent until examined with a low-power microscope. Dye penetrant revealed that the second crack was much more extensive, as shown in Fig. 10. No cracks were found in the bottom lug.

Opening the second crack and examining the fracture surface and microstructure revealed essentially identical features to the fracture in the gin involved in the accident. The tendency toward intergranular fracture in the examined gins was so pronounced that physical damage to the castings (dings)

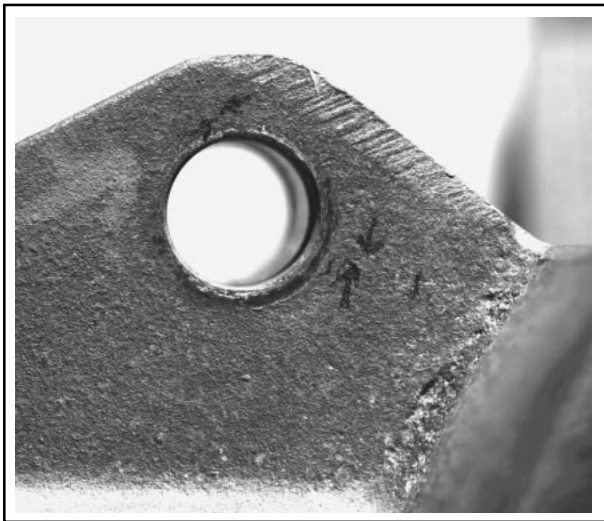


Fig. 9 A second cracked gin from service was discovered to have two small cracks in the top lug only. Arrows on the surface point to cracks.

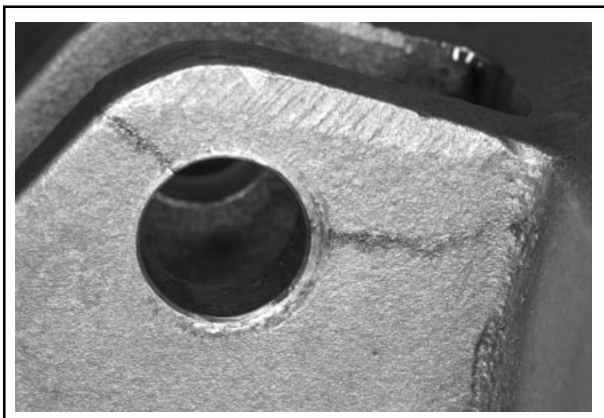


Fig. 10 Dye penetrant revealed extensive cracking in exemplar gin lug.

resulted in chipped-out pieces of metal by intergranular fracture instead of dents.

Mechanical Properties

Very little literature could be found on the 771-T7 alloy (also known as Precedent 71A). The alloy is an aluminum-zinc-magnesium heat treatable alloy. Table 1 provides the actual and specified chemistry. Two features stood out on inspection of the mechanical property data available in the literature:

- Tensile and yield strength increased in the over-aged T7 condition compared with the T6 temper.^[1]
- The alloy had a poor pressure tightness rating,^[2] which would indicate a potentially serious porosity problem. (A high pouring temperature was used to reduce the porosity.)

The base casting was made in a permanent mold, but the gating system had no evidence of design for progressive solidification. Dated product literature (no longer available) advertised good strength and toughness when solidification was directional (which would reduce porosity).

Tensile and compact-tension fracture toughness tests were performed on specimens cut from the base of both the casting involved in the accident and from exemplar castings. Because the wall thickness was generally 7.6 mm in the base, subsized specimens were required. Tensile specimens were square gage section and pin loaded (scaled to ASTM E 8 recommended dimensions). (All tensile and fracture toughness tests were performed by Advanced Technology Corporation in Oak Ridge, Tenn.) Ten-

Table 1 Actual and Standard Chemistry for 771 Aluminum Permanent Mold Alloy (AA-CS-M2-84)

Element	Actual casting	Aluminum Association standard
Silicon	0.024	0.15
Iron	<0.002	0.15
Copper	0.005	0.10
Manganese	0.010	0.10
Magnesium	0.682	0.8–1.0
Chromium	0.134	0.06–0.20
Nickel
Zinc	7.730	6.5–7.5
Titanium	0.161	0.10–0.20

Table 2 Miniature Tensile Test Results for the Accident Gin and Exemplars (35.6 mm Effective Gage Length)

Specimen	Source	Yield, MPa	Ultimate, MPa	Elongation, %
A1	Accident	322	337	0.7
A2	Accident	294	308	0.4
A3	Accident	292	309	0.6
A4	Accident	326	360	3.4
1A	Exemplar	278	286	0.4
1B	Exemplar	277	282	0.3
1C	Exemplar	285	304	0.2
1D	Exemplar	284	294	0.4
2A	Exemplar	276	307	2.3
2B	Exemplar	282	286	0.2
2C	Exemplar	273	298	0.2
3A	Same mold	266	282	0.8
3B	Different	302	316	1.2
3C	Foundry practice	309	319	0.9

Table 3 Fracture Toughness Results from the Accident Gin and Exemplar Gins Using 0.18T Disc Compact-Tension Specimens

Specimen	Source	a/w	a , mm	K_{Ic} , MPa \sqrt{m}
A	Accident	0.544	5.03	12.4
B	Accident	0.533	4.93	18.9
C	Accident	0.495	4.57	17.6
D	Accident	0.544	5.03	18.1
1B	Exemplar	0.607	5.61	13.2
1C	Exemplar	0.577	5.33	14.1
2B	Exemplar	0.567	5.23	12.0
2D	Exemplar	0.577	5.33	13.4
3B	Exemplar	0.514	4.75	9.68
3C	Exemplar	0.522	4.83	25.8?

sile test results are given in Table 2. The first three specimens from the accident gin, which exhibited low elongation values, had readily observable porosity on the fracture surfaces. The fourth specimen, A4, had 3.4% elongation and exhibited no observable porosity on the fracture surface at 10 to 20 \times magnification. Aluminum Association standards for cast alloys require a minimum tensile coupon elongation of 2.0% for the T71 temper. However, elongation for actual castings was not allowed to fall below 25% of that value, or 0.5%.^[3] Other exemplar gins exhibited similar mechanical property behavior.

Four 0.18T disc compact fracture toughness specimens from the accident castings and six from exemplar gins were tested according to ASTM E 1820. Porosity and low toughness made fatigue pre-

cracking very difficult. The crack front was not uniform and did not remain in the notch plane. The cracking appeared to follow an easy path through the microstructure. Fatigue pre-cracking was performed at 8.7 MPa \sqrt{m} . Valid plane-strain fracture toughness (K_{Ic}) results were not possible, but the plane-stress fracture toughness (K_c) numbers were measured and are shown in Table 3. The K_{Ic} results provide a minimum fracture toughness at a minimum thickness and are considered a material property. The K_c results from thinner materials are usually higher than K_{Ic} and can only apply to the thickness tested. These K_c results can be used as an upper bound on the toughness in the lugs. Actual K_{Ic} values for the lugs are probably slightly lower, if they could be measured.

The fatigue precracked region of one of the specimens was examined with an SEM for evidence of fatigue. Figure 11 shows both the fatigue and rupture regions; Fig. 12 and 13 show more detail of each. Fatigue was clearly by intergranular fracture, while the rupture region consisted of extensive casting void regions with intergranular fracture and some rupture by large microvoids. Microvoid coalescence was not observed anywhere near the hole walls in the actual gin failure.

Load Reconstruction

The issues of overloading and hole wall stresses were addressed by directly measuring hole wall forces using a custom clevis pin load cell made by Strainert of West Conshohocken, Pa. This load cell replaced



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the bolt attaching the ratchet to the pole gin base. Design and calibration of the load cell assumed uniform loading on the pin. It is not possible while on the pole to tighten the ratchet enough to keep the base tight against the pole. Therefore, the pole gin tilts down with load, so that the top lug is more heavily loaded than the bottom lug. Thus, the loads measured by the clevis pin cell are more of an average for both lugs.

A reconstruction of the loading arrangement using 2890 N (for a worst case) was made using a section of a wooden power pole, an exemplar pole gin and pulley, and load cells. The critical peak load position of the transformer was taken as the side pull required to clear the phone lines on the pole (load cell on tag line); this situation was common for the utility. The tag line pull was limited by the fact that two men pulling on the tag line were on a grass lawn after a

rain. It was estimated (by test) that both men could not pull more than 890 N together because of the loss of friction on the wet grass. Staying within the known confines of the pole and fenced-in backyard, a rigging condition was determined whereby the 2890 N transformer could be pulled clear of the phone lines. (The new, lighter transformer never got to this position.) This condition produced a reading of 6860 N on the clevis pin and a tag line load of 694 N. Statics calculations yielded a maximum download at the pulley of 6900 N, including a 10% friction contribution.

These conditions are reasonably within the 8900 N loading limit on the pole gin. Therefore, the pole gin was not overloaded at the time of failure. Checking with the electric utility about the type of operation and the transformers being used revealed little likelihood of prior overloading. Most of the transformers in their system were the same size as those involved in the accident. The few larger transformers were much larger and required heavy boom trucks for installation.

Abuse

The one factor still left to be considered was abuse. A pattern of tool marks was observed on the end piece that held the shackle. Figures 14 and 15 show two of the areas with these marks. Figure 15 reveals that the marks were a regularly spaced series of truncated pyramids. Some versions of framing hammers have this type of pattern, but the tips of the pyramids are never ground off. Sheetrock hammers

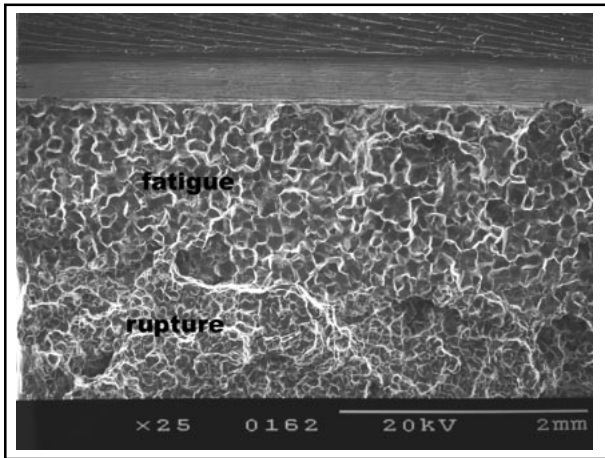


Fig. 11 Fatigue precrack and rupture region on one compact-tension fracture toughness specimen

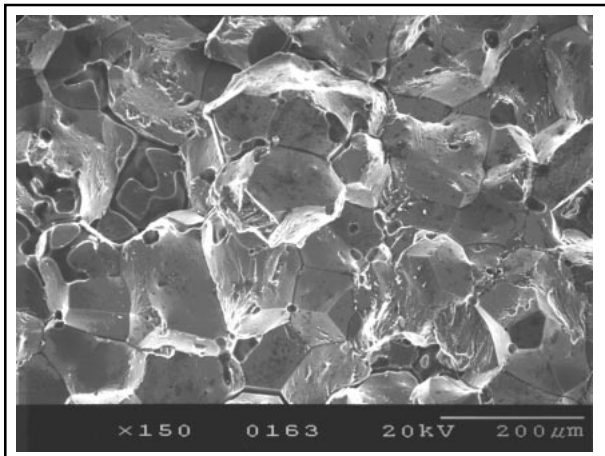


Fig. 12 No fatigue striations were observed in the intergranular fatigue area.

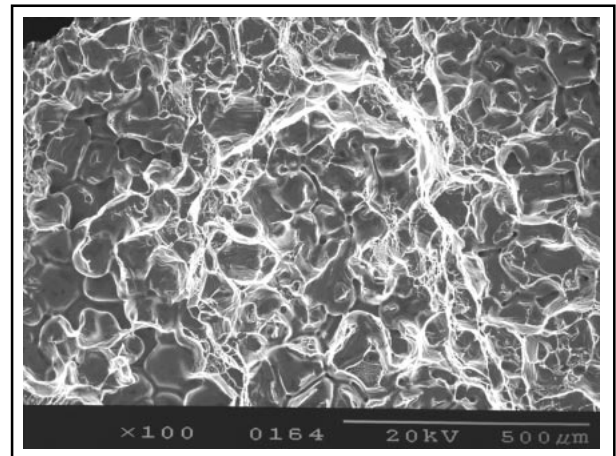


Fig. 13 The rupture region from the compact-tension specimen in Fig. 11 revealed extensive void area along with intergranular fracture and some microvoid rupture.

were found to have this truncated pyramid pattern on the striking face.

Calculations of the force required to create these marks were made by calculating the area of the dents and, knowing the hardnesses of the materials,^[4] yielded force estimates on the order of 8900 N. Withdrawal loads for a 16d box nail in a building stud are approximately 2890 N,^[5] so at least that much force must be generated by a hammer blow. Therefore, the use of this type of hammer and its effects on the lug failure had to be considered. The actual arrangement of the load path of the hammer blow on the end casting to the cracked lug hole argued for attenuation of hammer blow forces. The fiberglass rod between the end and base castings would attenuate some of the force. Significant attenuation was expected by the nylon belt, which

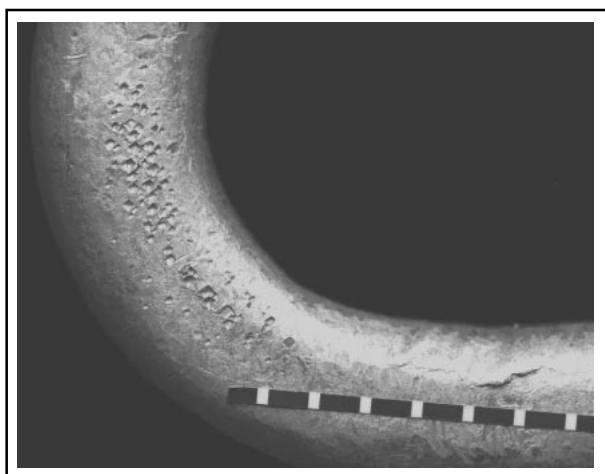


Fig. 14 Pyramid-shaped marks found on the eye for the pulley. (Scale marker is 5.1 mm/division.)

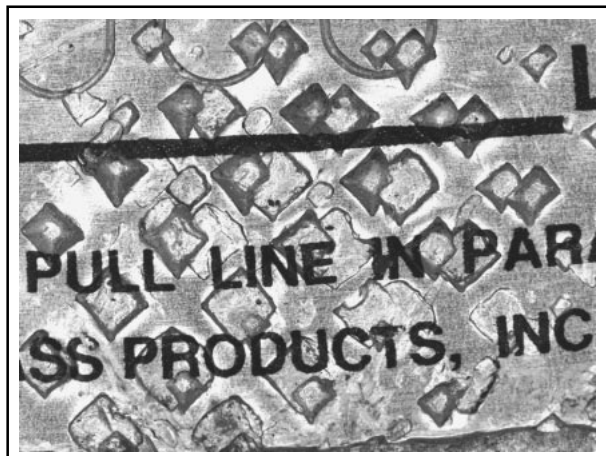


Fig. 15 The same pattern of truncated pyramids was found on a soft aluminum name plate.

was not snug to the pole. A recreation of the hammer blow to the gin was set up with an exemplar gin to determine the actual magnitude of the pin loads from a hammer strike.

An instrumented hammer was created by cutting off the striking face of a sheetrock hammer and attaching it to a steel rod. The total mass was approximately the same as the original hammer head. This arrangement permitted a full bridge axial force strain gage cell capable of measuring actual hammer forces to be created on the rod near the striking face, without the geometry problems resulting from gaging an actual hammer. Using an actual hammer face permitted comparison of the dents and thus the forces produced on the exemplar end casting to the actual dents in the accident gin in order to assure that hammer blows were comparable. The actual drop height was determined from tests on aluminum plates of similar hardness.

Belt tension in the nylon strap securing the ratchet to the pole was a major variable in terms of force attenuation in these tests. However, the ratchet mechanism was too coarse to provide much control over belt forces for low forces that were thought to be present. (It was difficult to tighten the ratchet above the lineman's head while on the pole.) Therefore, a shallow-tapered wooden wedge was inserted between the belt and the pole and driven to reach the desired belt preload from the instrumented clevis pin load cell. (No load was present on

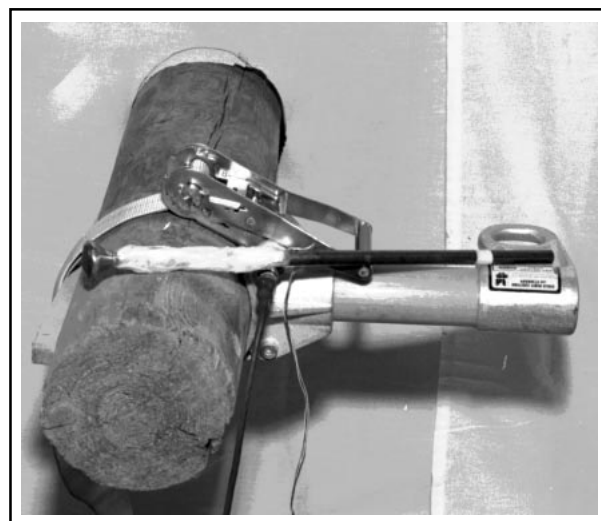


Fig. 16 Test setup for impact tests on the gin. The hammer load cell is lying on top of the gin and partially covers the clevis pin cell. The wedge used to adjust belt tension can be seen under the belt on the left side of the pole.



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the pulley.) Figure 16 shows the test setup, while Fig. 17 shows the striking face of the hammer. The wooden wedge can be seen under the belt on the opposite side of the pole from the gin.

A digital storage oscilloscope DSO-2102, made by Link Instruments of Fairfield, N.J., was used to record the data from the clevis pin cell and the instrumented bar/hammer. This system was capable of sampling at rates up to 1×10^8 samples/second. Actual data were recorded at 250 kHz. Strain gages were powered by Vishay 2310 (Vishay Measurements Group, Inc., Raleigh, N.C.) signal conditioners. (Evaluation of output on the conditioners revealed a level output voltage for the conditions used.)

Several tests were run at a drop height of 45.7 cm (determined to be representative of actual conditions), with varying belt tension loads. The bar/hammer, weighing 9.35 N, consistently produced a reading on the order of 8900 N force on the casting. The pin cell load did vary with belt forces, especially at the low forces that would have existed when readjusting the gin. Table 4 provides a summary of the load data. Figure 18 provides a sample set of output data in graphical form. Because belt tension was probably on the order of 44.5 to 223 N at the time of the accident, lug hole forces were only a fraction of the total forces (6860 N by measurement) encountered in lifting the transformer. Therefore, while clear evidence for hammer marks and thus improper care of the equipment was present, hammer blows were not significant in terms of ultimately

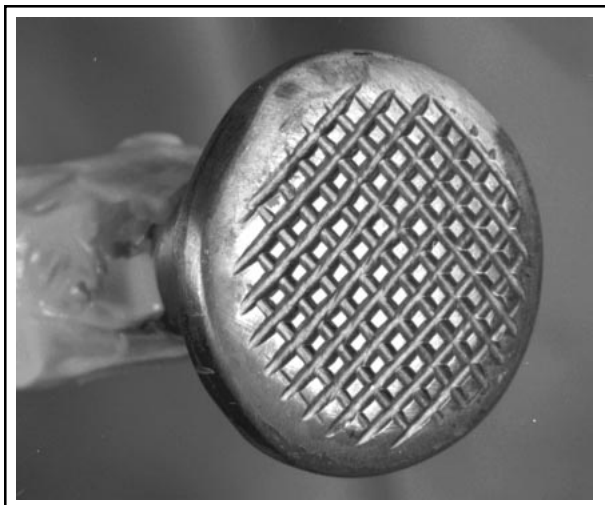


Fig. 17 Truncated pyramid pattern on the striking face of the Sheetrock hammer

cracking the lug holes. In fact, the cracked exemplar pole gin had less evidence of use and wear than the one involved in the accident. (Other gins were found cracked in earlier recalls.)

Stress Analysis

Conventional stress analysis using published eye bar procedures^[6] revealed stresses on the order of the yield strength at design load. While the lug hole is not in a classic eye bar design, this is a reasonable first approach to order-of-magnitude design stresses. Design stresses at this level would cause concern for fatigue in any aluminum alloy. Stresses at this level for this alloy would be expected to result in cracking, as was confirmed in a recall of the gin.

Discussion of Results

Aluminum alloy 771-T7 is not commonly used in manufacturing. The only experience the manufacturer of the pole gins had in using the alloy was with the same base used in a different application having very limited service loads. This alloy allowed the base to pass a 22,300 N proof load test. The 771-T7 alloy was in a peak strength heat treatment condition (even with a T7 temper) and in a composition range known for problems with stress-corrosion cracking (SCC). One data compilation gives the alloy a low rating for SCC.^[7] It is also known that moisture in the air can initiate and propagate SCC in 7000-series alloys.^[8] The role of SCC in the failure is difficult to identify with fractographic evidence; however, the relative humidity was high on the day of the accident. Additionally, the time at stress levels adequate to reach the local yield stress was several minutes for each load condition. Thus, there is likely an SCC contribution to the crack growth. Overaged alloys of this type also have a tendency to fracture in an intergranular mode

Table 4 Lug Hole Loads as a Function of Nylon Strap Tension for Impact Loads

Test	Hammer force, N	Lug hole force, N	Belt tension, N
a	8900	890	44.5
b	9790	2220	222
c	12,000	2220	476
d	11,100	3120	890
e	10,700	4000	1780

without SCC, as seen in the tensile test.^[9-11] Metallographic evaluation suggested base castings were cooled slowly after solution annealing and tempering, which would promote intergranular fracture.^[9,11] The combination of alloy selection and processing for the base casting resulted in a component that was very susceptible to subcritical crack growth and catastrophic failure.

Failure of the gin while lifting a transformer that was lighter than the one just lowered appears to be an unusual situation. It was originally thought that a small hole wall fatigue crack, which was destroyed when the bolt pulled out at fracture, could account for the failure. Thus, fracture toughness data were sought. However, the later discovery of a deeply cracked recalled gin and records of other cracked gins in recall revealed the importance of the issue of pin loading. The stress intensity for the vast majority of cracks increases as the square root of the crack length. A bolt-loaded hole produces a condition in which the stress intensity varies as the inverse square root of the crack length.^[12,13] The stress intensity drops as the crack length gets larger for a given load. A growing crack can self-arrest in this type of stress field and clearly did in this device. The elastic stress concentration from the hole wall also contributed to produce an even greater steep stress-intensity gradient that was adequate to arrest crack growth in a brittle alloy.

Crack growth under relatively static load, that is,

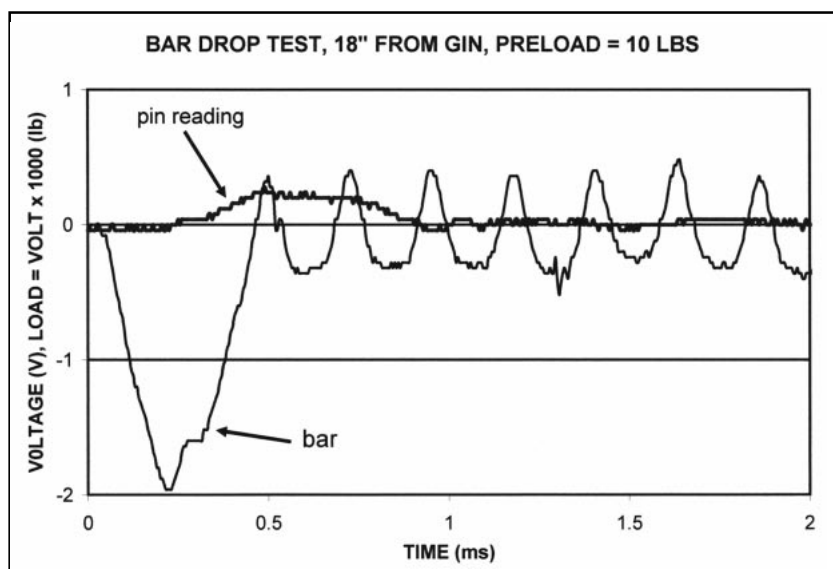


Fig. 18 Output from the clevis pin load cell for the case with 222 N belt tension. This was probably the maximum tension possible for the known conditions.

lifting the second gin, had to occur as a result of one or both of the SCC and creep-rupture mechanisms. Thus, a mechanism for a rising stress intensity for constant transformer weight did exist. Total rupture occurred as the linemen started to pull the transformer to the side to clear phone lines on the pole. Overall forces increased at this point. Thus, crack growth during the final lift plus the side pull finally triggered rupture.

Conclusions

The pole gin was not subject to service overloading at the time of failure. Previous overloading was unlikely because of the fact that there were few larger transformers in the power company system, and they were much larger and would require different rigging. Prior overloading was not required to initiate cracking when the nature of the alloy, its properties, porosity, and hole wall stresses were considered. The design itself was not suitable for long-term use. This is supported by the number of cracked gins discovered during a recall campaign. (The gin was taken out of service by the company.)

The 771-T7 was a poor choice for a casting in an application where failure could result in catastrophic consequences. The alloy selection was poor because of susceptibility to SCC and difficulties with porosity control. Extensive microporosity did weaken the casting.

Design stress levels were too high for an alloy of such low ductility. Stress levels exceeding the elastic limit in the hole wall will degrade fatigue life in aluminum alloys. These stress levels, when applied to the alloy 771-T7, created a high probability for crack initiation and growth, as evidenced by inspection of recalled gins.

References

1. J.G. Kaufman: *Introduction to Aluminum Alloys and Tempers*, ASM International, Materials Park, OH, 2000, p. 51.
2. J.E. Hatch, ed.: *Aluminum Properties and Physical Metallurgy*, American Society for Metals, Metals Park, OH, 1984, p. 345.
3. *Standards for Aluminum Sand and Permanent Mold Castings*, The Aluminum Association, 2000, AA-CS-M3-2000.



Failure Analysis of a Pole Gin (continued)

4. A.G. Atkins and D.K. Felbeck: "Applying Mutual Indentation Hardness Phenomena to Service Failure," *Met. Eng. Q.*, May 1974, pp. 364-70.
5. *The Encyclopedia of Wood*, Sterling Publishing Company, New York, NY, 1989, p. 7-2.
6. A. Blake: Chapter 29 in *Practical Stress Analysis in Engineering Design*, Marcel Dekker, New York, NY, 1982.
7. R.H. Jones, ed.: "Stress-Corrosion Cracking of Aluminum Alloys," *Stress-Corrosion Cracking*, American Society for Metals, Metals Park, OH, 1992, pp. 233-50.
8. M.O. Speidel: "Stress Corrosion Cracking of Aluminum Alloys," *Metall. Trans. A*, April 1975, 6, pp. 631-51.
9. D.S. Thompson: "Metallurgical Factors Affecting High Strength Aluminum Alloys Production," *Metall. Trans. A*, April 1975, 6, pp. 671-83.
10. G.T. Hahn and A.R. Rosenfield: "Metallurgical Factors Affecting Fracture Toughness of Aluminum Alloys," *Metall. Trans. A*, April 1975, 6, pp. 653-67.
11. M. Fine: "Precipitation Hardening of Aluminum Alloys," *Metall. Trans. A*, April 1975, 6, pp. 625-30.
12. J.M. Barson and S.T. Rolfe: Chapter 2 in *Fracture and Fatigue Control in Structures*, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 1987.
13. H. Tada, P.C. Paris, and G.R. Irwin: *The Stress Analysis of Cracks Handbook*, 3rd ed., American Society of Mechanical Engineers, New York, NY, 2000, p. 66.